

# Minimal Model for Neutrino Masses and Mixings

Paul H. Frampton

---

Talk at Second Tropical Conference  
on Particle Physics and Cosmology,  
May 4, 2000. San Juan, Puerto Rico.

---

# Abstract

Working in the framework of three chiral neutrinos with Majorana masses, we investigate a scenario first realized in an explicit model by Zee: that the neutrino mass matrix is strictly off-diagonal in the flavor basis, with all its diagonal entries precisely zero. This CP-conserving ansatz leads to two relations among the three mixing angles  $(\theta_1, \theta_2, \theta_3)$  and two squared mass differences. We impose the constraint  $|m_3^2 - m_2^2| \gg |m_2^2 - m_1^2|$  to conform with experiment, which requires the  $\theta_i$  to lie nearby one of four 1-parameter domains in  $\theta$ -space. We exhibit the implications for solar and atmospheric neutrino oscillations in each of these cases. A unique version of the Zee *ansatz* survives confrontation with experimental data, one which necessarily involves maximal just-so vacuum oscillations of solar neutrinos.

# Contents

1. Introduction	3
2. The Situation of the Data	3
3. Extending the Standard Model	3
4. The Logical Possibilities	4
5. Acknowledgement	5

# 1. Introduction

In this talk I will describe a paper published last year[1] with Shelly Glashow; the work was done at the Korean Institute for Advanced Study, Seoul, in June 1999 and the topic is what is the simplest way to extend the minimal standard model, where neutrinos are by definition massless, to accommodate the compelling evidence especially from the SuperKamiokande experiment for non-zero neutrino mass.

## 2. The Situation of the Data

The minimal standard model involves three chiral neutrino states, but it does not admit renormalizable interactions that can generate neutrino masses. Nevertheless, experimental evidence suggests that both solar and atmospheric neutrinos display flavor oscillations, and hence that neutrinos do have mass. Two very different neutrino squared-mass differences are required to fit the data:

$$10^{-11}\text{eV}^2 \leq \Delta_s \leq 10^{-5}\text{eV}^2 \quad \text{and} \quad \Delta_a \simeq 10^{-3}\text{eV}^2 \quad (1)$$

where the neutrino masses  $m_i$  are ordered such that:

$$\Delta_s \equiv |m_2^2 - m_1^2| \text{ and } \Delta_a \equiv |m_3^2 - m_2^2| \simeq |m_3^2 - m_1^2|$$

and the subscripts  $s$  and  $a$  pertain to solar and atmospheric oscillations respectively. The large uncertainty in  $\Delta_s$  reflects the several potential explanations of the observed solar neutrino flux: in terms of vacuum oscillations or large-angle or small-angle MSW solutions, but in every case the two independent squared-mass differences must be widely spaced with

$$r \equiv \Delta_s/\Delta_a < 10^{-2}$$

In a three-family scenario, four neutrino mixing parameters suffice to describe neutrino oscillations, akin to the four Kobayashi-Maskawa parameters in the quark sector.<sup>1</sup>

Solar neutrinos may exhibit an energy-independent time-averaged suppression due to  $\Delta_a$ , as well as energy-dependent oscillations depending on  $\Delta_s/E$ . Atmospheric neutrinos may exhibit oscillations due to  $\Delta_a$ , but they are almost entirely unaffected by  $\Delta_s$ . It is convenient to define neutrino mixing angles as follows:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = X \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

where

$$X = \begin{pmatrix} c_2 c_3 & c_2 s_3 & s_2 e^{-i\delta} \\ +c_1 s_3 + s_1 s_2 c_3 e^{i\delta} & -c_1 c_3 - s_1 s_2 s_3 e^{i\delta} & -s_1 c_2 \\ +s_1 s_3 - c_1 s_2 c_3 e^{i\delta} & -s_1 c_3 - c_1 s_2 s_3 e^{i\delta} & +c_1 c_2 \end{pmatrix}$$

<sup>1</sup>Two additional convention-independent phases are measurable in principle, but they ordinarily do not affect neutrino oscillations[2, 3]

with  $s_i$  and  $c_i$  standing for sines and cosines of  $\theta_i$ . For neutrino masses satisfying Eq.(1) the vacuum survival probability of solar neutrinos is[4]

$$P(\nu_e \rightarrow \nu_e)|_s \simeq 1 - \frac{\sin^2 2\theta_2}{2} - \cos^4 \theta_2 \sin^2 2\theta_3 \sin^2(\Delta_s R_s/4E) \quad (2)$$

whereas the transition probabilities of atmospheric neutrinos are:

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_\tau)|_a &\simeq \sin^2 2\theta_1 \cos^4 \theta_2 \sin^2(\Delta_a R_a/4E) \\ P(\nu_e \leftrightarrow \nu_\mu)|_a &\simeq \sin^2 2\theta_2 \sin^2 \theta_1 \sin^2(\Delta_a R_a/4E) \\ P(\nu_e \rightarrow \nu_\tau)|_a &\simeq \sin^2 2\theta_2 \cos^2 \theta_1 \sin^2(\Delta_a R_a/4E) \end{aligned} \quad (3)$$

None of these probabilities depend on  $\delta$ , the measure of CP violation.

## 3. Extending the Standard Model

Let us turn to the origin of neutrino masses. Among the many renormalizable and gauge-invariant extensions of the standard model that can do the trick are:

- The introduction of a complex triplet of mesons ( $T^{++}$ ,  $T^+$ ,  $T^0$ ) coupled bilinearly to pairs of lepton doublets. They must also couple bilinearly to the Higgs doublet(s) so as to avoid spontaneous  $B - L$  violation and the appearance of a massless and experimentally excluded majoron. This mechanism can generate an arbitrary complex symmetric Majorana mass matrix for neutrinos.
- The introduction of singlet counterparts to the neutrinos with very large Majorana masses. The interplay between these mass terms and those generated by the Higgs boson—the so-called see-saw mechanism—yields an arbitrary but naturally small Majorana neutrino mass matrix.
- The introduction of a charged singlet meson  $f^+$  coupled antisymmetrically to pairs of lepton doublets, *and* a doubly-charged singlet meson  $g^{++}$  coupled bilinearly both to pairs of lepton singlets and to pairs of  $f$ -mesons. An arbitrary Majorana neutrino mass matrix is generated in two loops.
- The introduction of a charged singlet meson  $f^+$  coupled antisymmetrically to pairs of lepton doublets *and* (also antisymmetrically) to a pair of Higgs doublets. This simple mechanism was first proposed by Tony Zee[5] and results (at one loop) in a Majorana mass matrix in the flavor basis ( $e, \mu, \tau$ )

of a special form:

$$\mathcal{M} = \begin{pmatrix} 0 & m_{e\mu} & m_{e\tau} \\ m_{e\mu} & 0 & m_{\mu\tau} \\ m_{e\tau} & m_{\mu\tau} & 0 \end{pmatrix} \quad (4)$$

In extending the standard model one crucial requirement is anomaly cancellation (*e.g.* [6]) and this is satisfied by all the above four scenarios. We focus here on the last scenario. In particular, we adopt the Zee *ansatz* for  $\mathcal{M}$  without committing ourselves to the Zee mechanism for its origin. Related discussions of Eq.(4) appear elsewhere[2, 7] The present work is essentially a continuation of [2].

Because the diagonal entries of  $\mathcal{M}$  are zero, the amplitude for no-neutrino double beta decay vanishes at lowest order [5] and this process cannot proceed at an observable rate. Furthermore, the parameters  $m_{e\mu}$ ,  $m_{e\tau}$  and  $m_{\mu\tau}$  may be taken real and non-negative without loss of generality, whence  $\mathcal{M}$  becomes real as well as traceless and symmetric. With this convention, the analog to the Kobayashi-Maskawa matrix becomes orthogonal:  $\mathcal{M}$  is explicitly CP invariant and  $\delta = 0$ . But as we have noted, it is well known[4] that the mere existence of a squared-mass hierarchy virtually precludes any detectable manifestation of CP violation in the neutrino sector.

The sum of the neutrino masses (the eigenvalues of  $\mathcal{M}$ ) vanishes:

$$m_1 + m_2 + m_3 = 0 \quad (5)$$

An important result emerges when the squared-mass hierarchy Eqs.(1) is taken into account along with Eq.(5). In the limit  $r \rightarrow 0$ , two of the squared masses must be equal.

## 4. The Logical Possibilities

There are two possibilities. In case A, we have  $m_1 + m_2 = 0$  and  $m_3 = 0$ . This case arises iff at least one of the three parameters in  $\mathcal{M}$  vanishes. In case B, we have  $m_1 = m_2$  and  $m_3 = -2m_2 > 0$ . This case arises iff the three parameters in  $\mathcal{M}$  are equal to one another. Of course,  $r$  is small but it does not vanish: in neither case can the above relations among neutrino masses be strictly satisfied. But they must be nearly satisfied. Consequently we may deduce certain approximate but useful restrictions on the permissible values of the neutrino mixing angles  $\theta_i$ . Prior to examining these restrictions, we note that Eqs.(5) and (1) exclude the possibility that the three neutrinos are nearly degenerate in mass. If the Zee *ansatz* is even approximately realized in nature, no neutrino mass can exceed a small fraction of an electron volt in magnitude and neutrinos are unlikely to contribute significantly to the dark matter of the universe.

We first consider case A. The relation  $m_1 + m_2 = 0$  may be obtained in three ways depending on which

parameter in  $\mathcal{M}$  is set to zero. If  $m_{e\mu} = 0$ , the quantum number  $L_\tau - L_e - L_\mu$  is conserved. It follows that  $\cos\theta_1 = 0$  and  $\theta_3 = \pi/4$ . We see from Eq.(3) that atmospheric  $\nu_\mu$ 's oscillate exclusively into  $\nu_e$ 's and *vice versa*. This subcase (or any nearby assignment of mixing angles) is strongly disfavored by SuperKamiokande data[8].

Alternatively we may set  $m_{e\tau} = 0$  to obtain conservation of  $L_\mu - L_\tau - L_e$ . For this subcase, we obtain  $\sin\theta_1 = 0$  and  $\theta_3 = \pi/4$ . We see from Eq.(3) that atmospheric  $\nu_\mu$ 's do not oscillate at all. This subcase (or any nearby assignment of mixing angles) is also strongly disfavored by SuperKamiokande data[8].

The last and best ([2]) version of Case A has  $m_{\mu\tau} = 0$  and leads to conservation of  $L_e - L_\mu - L_\tau$ . For this subcase, we obtain  $\sin\theta_2 = 0$  and  $\theta_3 = \pi/4$ . We see from Eq.(2) that solar neutrino oscillations are maximal:

$$P(\nu_e \rightarrow \nu_e)|_s = 1 - \sin^2(\Delta_s R_s/4E) \quad (6)$$

Moreover, we see from Eq.(3) that atmospheric  $\nu_\mu$ 's oscillate exclusively into  $\nu_\tau$ 's with the unconstrained mixing angle  $\theta_1$ :

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_\tau)|_a &= \sin^2 2\theta_1 \sin^2(\Delta_a R_a/4E) \\ P(\nu_\mu \leftrightarrow \nu_e)|_a &= 0 \quad P(\nu_e \rightarrow \nu_\tau)|_a = 0 \end{aligned} \quad (7)$$

This implementation of the Zee *ansatz* is compatible with experiment: It predicts maximal solar oscillations (without an energy-independent term) and it is consistent with the just-so vacuum oscillation hypothesis[9]. However, it is evidently not compatible with the small-angle MSW explanation of solar neutrino data. Neither is it compatible with the large-angle MSW solution, because it predicts virtually maximal solar neutrino oscillations. If  $m_{\mu\tau}$  is permitted to depart slightly from zero so as to generate a small finite value of  $r \equiv \Delta_s/\Delta_a$ , the coefficient of the oscillatory term in Eq.(6) will depart from unity by a term of order  $r^2 \leq 10^{-4}$ . The resulting solar-neutrino oscillations remain nearly maximal: they are energy independent and experimentally disfavored unless  $\Delta_s$  lies within the just-so domain.

We furthermore note that the  $m_{\mu\tau} \simeq 0$  version of the Zee *ansatz* admits atmospheric neutrino oscillations of the type  $\nu_\mu \rightarrow \nu_\tau$  with any value of the mixing angle  $\theta_1$ . At the same time, it precludes all oscillations involving atmospheric  $\nu_e$ . These results are quite in accord with SuperKamiokande data.

Others have noted[10] that the relation  $m_1^2 = m_2^2$  is preserved by radiative corrections for case A. This is not necessarily true for case B, where the relations satisfied by the neutrino masses,  $m_3 = -2m_2 = -2m_1$ , are not a consequence of a symmetry principle. In any event, we argue that case B cannot fit the data. Ref.[2] shows that case B leads to the relation:

$$\tan^2 \theta_2 = 1/2 \quad (8)$$

Along with Eqs.(2), this implies:

$$P(\nu_e \rightarrow \nu_e)|_a = 1 - (8/9) \sin^2(\Delta_a R_a/4E) \quad (9)$$

That is, we must have large (almost maximal) oscillations of atmospheric  $\nu_e$ . This result is strongly disfavored by SuperKamiokande data[8], so that case B can be rejected without further ado.

Our conclusion is simple. We find that one (and only one!) of the realizations of the Zee *ansatz* incorporating a squared-mass hierarchy is compatible with both solar and atmospheric neutrino data. It corresponds to the assignments  $m_{e\mu} \simeq m \cos \theta_1$ ,  $m_{e\tau} \simeq m \sin \theta_1$  and  $m_{\mu\tau} \ll m$ , with  $\theta_2 \simeq 0$  and  $\theta_3 \simeq \pi/4$ . Near this domain, atmospheric electron neutrinos oscillate negligibly, while atmospheric muon neutrinos oscillate into tau neutrinos with the arbitrary mixing angle  $\theta_1$ . Solar neutrino oscillations are very nearly maximal. They can be described by vacuum oscillations, but not by MSW oscillations. It is straightforward to implement the Zee model so as to conserve  $L_e - L_\mu - L_\tau$  *exactly* so as to obtain  $m_{\mu\tau} = 0$  to all orders[10]. Of course, this must not be done! Some unspecified new physics (beyond the introduction of Zee's  $f^+$  meson) is required to lift the degeneracy between  $m_1^2$  and  $m_2^2$  so as to yield an extreme hierarchy of neutrino squared-mass differences, with  $\Delta_s \sim 10^{-8} \Delta_a$ .

## 5. Acknowledgement

This work was supported in part by the U.S. Department of Energy under grant number DE-FG02-97ER-41036.

## References

- [1] P.H. Frampton and S.L. Glashow, Phys. Lett. **B461**, 95 (1999).
- [2] C. Jarlskog *et al.*, Phys. Lett. **B449**, 240 (1999);  
M. Matsuda *et al.*, **hep-ph/0005147**.
- [3] S.L. Glashow, in Atti dei Convegni Lincei, Rome **133**, 69 (1997).
- [4] H. Georgi and S.L. Glashow, **hep-th/9808293**
- [5] A. Zee, Phys. Lett. **93B**, 389 (1980).
- [6] P.H. Frampton and S.L. Glashow, Phys. Lett. **B190**, 157 (1987); Phys. Rev. Lett. **58**, 2168 (1987).
- [7] A. Yu. Smirnov and Zhijian Tao, Nucl. Phys. B426 (1994) 415;  
A. Yu. Smirnov and M. Tanimoto, Phys. Rev. D55 (1997) 1665.

- [8] T. Kajita for the SuperKamiokande collaboration, **hep-ex/9807003**.
- [9] V. Barger, K. Whisnant and R.J.N. Phillips, Phys. Rev. D24 (1981) 538;  
S.L. Glashow and L.M. Krauss, Phys. Lett. B190 (1987) 199.
- [10] R. Barbieri, G.G. Ross and A. Strumia, **hep-ph/9906470**.